MATHEMATICS - 1998

PART - A

Directions : *Read questions 1 to 40 carefully and choose from amongst the alternatives given below each question the correct lettered choice(s). A question may have ONE OR MORE correct alternatives. In order to secure any marks for a given question, ALL correct lettered alternative(s) must be chosen.*

1. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals :

(A) $128\,\omega$ (B) $-128\,\omega$

(C) $128\,\omega^2$ (D) $-128\,\omega^2$.

2. Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, \ldots$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$

(C) 1 (D) 0

3. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is :

(A) at least 30 (B) at most 20

(C) exactly 25 (D) none of the above

4. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a :

(A) rectangle (B) square

(C) cyclic quadrilateral (D) rhombus.

5. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is :

(A) 0 (B) 1

(C) 3 (D) 4

6. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is :

(A) 1 (B) 2

(C) 0 (D) $\frac{1}{2}$

7. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals :

(A) 8 (B) 6

(C) 10 (D) 12

8. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then :

(A) $a = 2, b = 4$ (B) $a = 3, b = 4$

(C) $a = 2, b = 3$ (D) $a = 3, b = 5$

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then :

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$

(C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

10. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :

(A) $\frac{13}{32}$ (B) $\frac{1}{4}$

(C) $\frac{1}{32}$ (D) $\frac{3}{16}$

11. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals :

(A) i (B) $i - 1$

(C) $-i$ (D) 0

12. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is :

(A) 0 (B) 1

(C) 2 (D) infinite

13. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f :

(A) does not exist because f is unbounded.

(B) is not attained even though f is bounded

(C) is equal to 1

(D) is equal to -1

14. Number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240 is :

(A) 4 (B) 8

(C) 10 (D) 3

15. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$:

(A) exists and it equals $\sqrt{2}$

(B) exists and it equals $-\sqrt{2}$

(C) does not exist because $x - 1 \to 0$

(D) does not exist because left hand limit is not equal to right hand limit

16. If in a triangle PQR, $\sin P$, $\sin Q$, $\sin R$ are in A. P., then :

(A) the altitudes are in A. P.
(B) the altitudes are in H. P.
(C) the medians are in G. P.
(D) the medians are in A. P.

17. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals :

(A) $(n-1)a_n$
(B) na_n
(C) $\frac{1}{2} na_n$
(D) None of the above

18. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational point(s).

(A) centroid
(B) incentre
(C) circumcentre
(D) orthocentre

(A rational point is a point both of whose co-ordinates are rational numbers)

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is :

(A) 0
(B) 1
(C) 2
(D) infinite

20. If $x > 1, y > 1, z > 1$ are in G. P., then $\frac{1}{1 + \ln x}, \frac{1}{1 + \ln y}, \frac{1}{1 + \ln z}$ are in :

(A) A.P.
(B) H.P.
(C) G.P.
(D) None of the above

21. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3 \sin^2 x - 7 \sin x + 2 = 0$ is :

(A) 0
(B) 5
(C) 6
(D) 10

22. The order of the differential equation whose general solution is given by $y = (C_1 + C_2) \cos (x + C_3) - C_4 e^{x + C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is :

(A) 5
(B) 4
(C) 3
(D) 2

23. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then :

(A) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
(B) $f(x) = \sin x, g(x) = |x|$
(C) $f(x) = x^2, g(x) = \sin \sqrt{x}$
(D) f and g cannot be determined

24. Let $A_0 A_1 A_2 A_3 A_4 A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments $A_0 A_1$, $A_0 A_2$ and $A_0 A_4$ is :

(A) $\frac{3}{4}$
(B) $3\sqrt{3}$
(C) 3
(D) $\frac{3\sqrt{3}}{2}$

25. For three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) \cdot \vec{u}$

(C) $\vec{v} \cdot (\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

26. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is :

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

27. Let $h(x) = \min\{x, x^2\}$, for every real number of x. Then :

(A) h is continuous for all x

(B) h is differentiable for all x

(C) $h'(x) = 1$, for all $x > 1$

(D) h is not differentiable at two values of x

28. If $f(x) = 3x - 5$, then $f^{-1}(x)$:

(A) is given by $\frac{1}{3x-5}$

(B) is given by $\frac{x+5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto.

29. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then.

(A) $P(E/F) + P(\overline{E}/F) = 1$ (B) $P(E/F) + P(E/\overline{F}) = 1$

(C) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (D) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

30. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then :

(A) $x = 3, y = 1$ (B) $x = 1, y = 3$

(C) $x = 0, y = 3$ (D) $x = 0, y = 0$

31. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :

(A) $\frac{1}{2}$ (B) $\frac{1}{32}$

(C) $\frac{31}{32}$ (D) $\frac{1}{5}$

32. An $n-$digit number is a positive number with exactly n digits. Nine hundred distinct $n-$digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is :

(A) 6 (B) 7

(C) 8 (D) 9

33. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals :

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$

(C) $\frac{2}{15}$ (D) $\frac{1}{3}$

34. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then :

(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$

(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 - 3n + 3$

35. Which of the following number(s) is/are rational ?

(A) $\sin 15°$ (B) $\cos 15°$

(C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

36. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then :

(A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$

(C) $x_1 x_2 x_3 x_4 = c^4$ (D) $y_1 y_2 y_3 y_4 = c^4$

37. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then :

(A) occurrence of $E \Rightarrow$ occurrence of F

(B) occurrence of $F \Rightarrow$ occurrence of E

(C) non-occurrence of $E \Rightarrow$ non-occurrence of F

(D) none of the above implications holds

38. Which of the following expressions are meaningful question

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

(C) $(\vec{u} \cdot \vec{v})\, \vec{w}$ (D) $\vec{u} \times (\vec{v} \cdot \vec{w})$

39. If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is :

(A) $\frac{1}{2}$ (B) 0

(C) 1 (D) $-\frac{1}{2}$

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then :

(A) h is increasing whenever f is increasing
(B) h is increasing whenever f is decreasing
(C) h is decreasing whenever f is decreasing
(D) nothing can be said in general.

ANSWERS

1. (D)	**2.** (C)	**3.** (C)	**4.** (D)	**5.** (B)	**6.** (A)
7. (C)	**8.** (C)	**9.** (D)	**10.** (A)	**11.** (B)	**12.** (A)
13. (D)	**14.** (A)	**15.** (D)	**16** (B)	**17.** (C)	**18.** (A)
19. (C)	**20.** (B)	**21.** (C)	**22.** (C)	**23.** (A)	**24.** (C)
25. (C)	**26.** (B)	**27.** (A), (C), (D)	**28.** (B)	**29.** (A), (D)	**30.** (D)
31. (A)	**32.** (B)	**33.** (B)	**34.** (B)	**35.** (C)	
36. (A), (B), (C), (D)		**37.** (D)	**38.** (A), (C)	**39.** (A)	**40.** (A), (C)

SOLUTIONS

1. $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$$= (-2\omega^2)^7 = (-2)^7 (\omega^2)^7 = -128 \cdot \omega^{14} = -128\,\omega^2$$

Therefore, (D) is the Ans.

2. Let $T_m = a + (m-1)\,d = \dfrac{1}{n}$

and $T_n = a + (n-1)\,d = \dfrac{1}{m}$

$\Rightarrow \quad (m-n)\,d = \dfrac{1}{n} - \dfrac{1}{m} = \dfrac{m-n}{mn} \quad \Rightarrow \quad d = \dfrac{1}{mn}$

Again $T_{mn} = a + (mn-1)\,d$

$$= a + (mn - n + n - 1)\,d$$

$$= a + (n-1)\,d + (mn - n)\,d$$

$$= T_n + n(m-1) \cdot \frac{1}{mn}$$

$$= \frac{1}{m} + \frac{(m-1)}{m} = \frac{1}{m} + 1 - \frac{1}{m} = 1$$

Therefore, (C) is the Ans.

3. Let number of newspaper which are read be n. Then

$$60n = 300 \times 5$$

$\Rightarrow \quad n = 25$

Therefore, (C) is the Ans.

4. Slope of $x + 3y = 4$ is $-1/3$

and slope of $6x - 2y = 7$ is 3.

therefore, these two lines are perpendicular which show that both diagonals are perpendicular. Hence *PQRS* must be a rhombus.

5. $x^2 \quad y^2 = 4$ (given)

centre $= c_1 \equiv (0, 0)$ and $R_1 = 2$.

Again $x^2 + y^2 - 6x - 8y - 24 = 0$ then $c_2 \equiv 3, 4$

and $\qquad R_2 - 7.$

Again $\qquad c_1 c_2 = 5 = R_2 - R_1$

Therefore, the given circles touch internally such that they can have just one common tangents at the point of contact.

Therefore, (B) is the Ans.

6. $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (x - [x])\,dx = \int_{-1}^{1} x\,dx - \int_{-1}^{1} [x]\,dx$

$= 0 - \int_{-1}^{1} [x]\,dx \qquad$ [$\because$ x is an odd function]

But $\quad [x] = \begin{cases} -1 & \text{if} \quad 1 \le x < 0 \\ 0 & \text{if} \quad 0 \le x < 1 \\ 1 & \text{if} \quad x = 1 \end{cases}$

$\therefore \int_{-1}^{1} [x]\,dx = \int_{-1}^{0} [x]\,dx + \int_{0}^{1} [x]\,dx$

$= \int_{1}^{0} (-1)\,dx + \int_{0}^{1} 0\,dx$

$= -[x]_{-1}^{0} + 0 = 1$

Thus $\int_{-1}^{1} f(x)\,dx = 1.$

Therefore, (A) is the Ans.

7. $16x^2 + 25y^2 = 400 \qquad$ (given)

$\Rightarrow \qquad \dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

Here $\quad a^2 = 25, \quad b^2 = 16$

But $\quad b^2 - a^2(1 - e^2)$

$\Rightarrow \qquad 16 = 25(1 - e^2) \quad \Rightarrow \quad \dfrac{16}{25} = 1 - e^2$

$\Rightarrow \qquad e^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

$\Rightarrow \qquad e = 3/5$

Now foci of the ellipse are $\pm ae, 0 = \pm 3, 0$

we have $3 = a \cdot \dfrac{3}{5} \qquad \Rightarrow \qquad a = 5$

Now $PF_1 + PF_2 =$ focal distance $= 2a = 2 \times 5 - 10$

Therefore, (C) is the Ans.

8. *PQRS* is a parallelogram if and only if the mid-point of *PR* is same as that if the mid-point of *QS*. That is, if and only if

$$\frac{1+5}{2} = \frac{4+a}{2} \quad \text{and} \quad \frac{2+7}{2} = \frac{6+b}{2}$$

$\Rightarrow$ $a = 2$ and $b = 3$. Therefore, (C) is the Ans.

9. It is given that $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent $\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0 \text{ Apply } C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

Now expanding along R_1,

$\Rightarrow$ $-(\beta - 1) = 0 \Rightarrow \beta = 1$

Also $|\vec{c}| = \sqrt{3}$ (given) where $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ (given)

$\Rightarrow$ $1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow$ $1 + \alpha^2 + 1 = 3 \rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

Therefore, (D) is the correct Ans.

10. P (2 white and 1 black) $= P(W_1 W_2 B_3 \text{ or } W_1 B_2 W_3 \text{ or } B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1) P(W_2) P(B_3) + P(W_1) P(B_2) P(W_3) + P(B_1)(W_2)(W_3)$

$= \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4}$

$= \frac{1}{32}(9 + 3 + 1) = \frac{13}{32}$

Therefore, (A) is the Ans.

11. $\sum_{i=1}^{13} (i^n + i^{n+1}) = \sum_{i=1}^{13} i^n (1 + i) = (1 + i) \sum_{i=1}^{13} i^n$

$= (1 + i)(i + i^2 + i^3 + \ldots i^{13}) = (1 + i) \left\{ \frac{i(1 - i)}{1 - i} \right\}$

$= (1 + i)\, i = -1 + i$, Therefore, (B) is the Ans.0

12. The maximum value of $f(x) = \cos x + \cos(\sqrt{2}x)$ is 2 which occurs at $x = 0$. Also, there is no value of x for which this value will be attained again.

Imp. note : This question can be solved by calculus also.

13. $f(x) = \frac{x^2 - 1}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$f(x)$ will be minimum when $\frac{2}{x^2 + 1}$ is maximum, i.e., when $x^2 + 1$ is minimum

i.e. at $x = 0$.

$\therefore$ Minimum value of $f(x)$ is $f(0) = -1$.

Therefore, (D) is the Ans.

$$\frac{1+5}{2}=\frac{4+a}{2} \quad \text{and} \quad \frac{2+7}{2}=\frac{6+b}{2}$$

$\Rightarrow \quad a = 2$ and $b = 3$. Therefore, (C) is the Ans.

9. It is given that $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent $\Rightarrow [\vec{a}\ .\ \vec{b}\ \ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha-1 & \beta-1 \end{vmatrix} = 0 \text{ Apply } C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

Now expanding along R_1,

$\Rightarrow \quad -(\beta-1)=0 \Rightarrow \quad \beta = 1$

Also $|\vec{c}| = \sqrt{3}$ (given) where $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ (given)

$\Rightarrow \quad 1+\alpha^2+\beta^2 = 3$

$\Rightarrow \quad 1+\alpha^2+1 = 3 \quad \to \quad \alpha^2 = 1 \quad \Rightarrow \quad \alpha = \pm 1$

Therefore, (D) is the correct Ans.

10. P (2 white and 1 black) $= P(W_1 W_2 B_3$ or $W_1 B_2 W_3$ or $B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1) P(W_2) P(B_3) + P(W_1) P(B_2) P(W_3) + P(B_1) (W_2) (W_3)$

$= \frac{3}{4}\cdot\frac{2}{4}\cdot\frac{3}{4} + \frac{3}{4}\cdot\frac{2}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}$

$= \frac{1}{32}(9+3+1) = \frac{13}{32}$

Therefore, (A) is the Ans.

11. $\sum_{i=1}^{13} (i^n + i^{n+1}) = \sum_{i=1}^{13} i^n (1+i) = (1+i) \sum_{i=1}^{13} i^n$

$= (1+i)(i+i^2+i^3+\ldots i^{13}) = (1+i)\left\{\frac{i(1-i)}{1-i}\right\}$

$= (1+i)\, i = -1 + i$, Therefore, (B) is the Ans.0

12. The maximum value of $f(x) = \cos x + \cos(\sqrt{2}x)$ is 2 which occurs at $x = 0$. Also, there is no value of x for which this value will be attained again.

Imp. note : This question can be solved by calculus also.

13. $f(x) = \frac{x^2-1}{x^2+1} = 1 - \frac{2}{x^2+1}$

$f(x)$ will be minimum when $\frac{2}{x^2+1}$ is maximum, i.e., when x^2+1 is minimum

i.e. at $x = 0$.

$\therefore$ Minimum value of $f(x)$ is $f(0) = -1$.

Therefore, (D) is the Ans.

19. For ellipse condition of tangency is $c^2 = a^2m^2 + b^2$

$\Rightarrow \quad c^2 = 4 \times 4 + 1 = 17$

$\Rightarrow \quad c = \pm\sqrt{17}$ therefore, (C) is the Ans.

20. Let the common ratio of the G.P. be r. Then

$$y = xr \quad \text{and} \quad z = xr^2$$

$\Rightarrow \ \ln y = \ln x + \ln r$ and $\ln z = \ln x + 2\ln r$

Putting $A = 1 + \ln x$, $D = \ln r$

Then $\dfrac{1}{1+\ln x} = \dfrac{1}{A}$, $\dfrac{1}{1+\ln y} = \dfrac{1}{1+\ln xr} = \dfrac{1}{1+\ln x+\ln r} = \dfrac{1}{A+D}$

and $\dfrac{1}{1+\ln z} = \dfrac{1}{1+\ln x+2\ln r} = \dfrac{1}{A+2D}$

Therefore, $\dfrac{1}{1+\ln x}, \dfrac{1}{1+\ln y}, \dfrac{1}{1+\ln z}$ are in H.P.

So (B) is the Ans.

21. $3\sin^2 x - 7\sin x + 2 = 0$ (given)

$\Rightarrow \quad 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow \quad 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow \quad (3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \quad \sin x = 1/3$ or $\sin x = 2$ (sin x = 2 is rejected).

$\Rightarrow \quad x = n\pi + (-1)^n \sin^{-1}\dfrac{1}{3}, \ n \in I$

For $0 \le n \le 5$, $x \in [0, 5\pi]$

$\therefore$ There are six values of $x \in [0, 5\pi]$ which satisfy the equation

$$3\sin^2 x - 7\sin x + 2 = 0$$

Therefore, (C) is the Ans.

22. $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$ (given) ...(1)

$\Rightarrow \quad y = (c_1 + c_2)\cos(x + c_3) - c_4 e^x \cdot e^{c_5}$

Now let $c_1 + c_2 = A$, $c_3 = B$, $c_4 e^{c_5} = C$

$\Rightarrow \quad y = A\cos(x + B) - Ce^x$ ...(2)

differentiate w.r.t. x

$\Rightarrow \quad \dfrac{dy}{dx} = -A\sin(x + B) - Ce^x$ ...(3)

differentiate again w.r.t x

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x$ ...(4)

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -y - 2Ce^x$ from (2) ...(5)

$\Rightarrow \quad \dfrac{d^2y}{dx^2} + y = -2Ce^x$

differentiate again w.r.t. x

$\Rightarrow \quad \dfrac{d^3y}{dx^3} + \dfrac{dy}{dx} = -2Ce^x$ ...(6)

$$\Rightarrow \qquad \frac{d^3y}{dx^3} + \frac{dy}{dx} = \frac{d^2y}{dx^2} \qquad \text{from (5)}$$

which is a differential equation of order 3. Therefore, (C) is the Ans.

23. Let $f(x) = \sin^2 x$ and $g(x) = \sqrt{x}$

Now $fog(x) = f(g(x)) = f(\sqrt{x}) = \sin^2 \sqrt{x}$

and $gof(x) = g(f(x)) = g(\sin^2 x) = \sqrt{\sin^2 x} = |\sin x|$

again, $f(x) = \sin x$, $g(x) = |x|$

$fog(x) = f[g(x)] = f(|x|) = \sin|x| \neq (\sin \sqrt{x})^2$

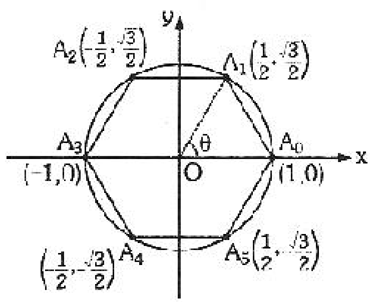
When $f(x) = x^2$, $g(x) = \sin\sqrt{x}$

$fog(x) = f[g(x)] = f(\sin\sqrt{x}) = (\sin\sqrt{x})^2$

and $(gof)(x) = g[f(x)] = g(x^2) = \sin\sqrt{x^2} = \sin|x| \neq |\sin x|$

Therefore, (A) is the Ans.

24.



$$A_0A_1^2 = \left(1 - \frac{1}{2}\right)^2 + \left(0 - \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$\Rightarrow \qquad A_0A_1 = 1$

Next, $A_0A_2^2 = \left(1 + \frac{1}{2}\right)^2 + \left(0 - \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4} + \frac{3}{4}$

$= \frac{12}{4} = 3$

$\Rightarrow \qquad A_0A_2 = \sqrt{3}$

Next $A_0A_4^2 = \left(1 + \frac{1}{2}\right)^2 + \left(0 + \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{4}\right) = \frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3$

$\Rightarrow \qquad A_0A_4 = \sqrt{3}$

Thus, $(A_0A_1)(A_0A_2)(A_0A_4) = 3$.

25. $[\underline{m}\ \underline{n}\ \underline{a}] = -[\underline{a}\ \underline{n}\ \underline{m}] = [\underline{n}\ \underline{m}\ \underline{a}] = [\underline{m}\ \underline{a}\ \underline{n}]$

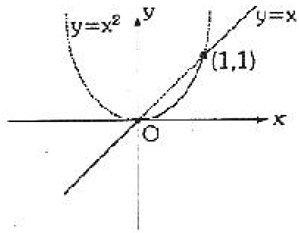
Therefore, (C) is the Ans.

26. The probability that only two tests are needed = probability that the first machine tested is faulty × (probability that the second machine tested is faulty given that the first machine tested is faulty $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$. Therefore, (B) is the Ans.

27. $h(x) = \min\{x, x^2\}$ (given).

We will trace $h(x) = x$ and $h(x) = x^2$ separately.

From fig it is clear that



$$h(x) = \begin{cases} x, & \text{if} \quad x \le 0 \\ x^2, & \text{if} \quad 0 \le x \le 1 \\ x, & \text{if} \quad x \ge 1 \end{cases}$$

From the graph it is clear that h is continuous for all $x \in R$ and $h'(x) = 1$ for all $x > 1$ and h is not differentiable at $x = 0$ and $x = 1$. Therefore, (A), (C) and (D) are the answers.

28. $f(x) = 3x - 5$ (given)

Let $y = f(x) = 3x - 5 \Rightarrow y + 5 = 3x \Rightarrow x = \dfrac{y+5}{3}$ ...(1)

and $y = f(x) \Rightarrow x = f^{-1}(y)$ ...(2)

From (1) and (2)

$$f^{-1}(y) = \frac{y+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}$$

Therefore, (B) is the Ans.

29. **A.** $P(E/F) + P(\bar{E}/F) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(\bar{E} \cap F)}{P(F)}$

$$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)}$$

$= \dfrac{P(F)}{P(F)} = 1$ therefore, A is the Ans.

B. $P(E/F) + P(E/\bar{F}) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(E \cap \bar{F})}{P(\bar{F})}$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{1 - P(F)} \neq 1$$

Therefore, (B) is not the Ans.

C. $P(\bar{E}/F) + P(E/\bar{F}) = \dfrac{P(\bar{E} \cap F)}{P(\bar{F})} + \dfrac{P(E \cap F)}{P(\bar{F})}$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{1 - P(F)}$$

Therefore, (C) is not the Ans.

D. $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = \dfrac{P(E \cap \bar{F})}{P(\bar{F})} + \dfrac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$

$$= \frac{P(E \cap \bar{F}) + P(\bar{E} \cap \bar{F})}{P(\bar{F})}$$

$= \dfrac{P(\bar{F})}{P(\bar{F})} = 1$. Therefore, (D) is true.

So (A), (D) are the Ans.

30. $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ (given)

$\Rightarrow \quad -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & 1 \\ 20 & i & i \end{vmatrix} = 0$ [$\Rightarrow C_2$ and C_3 are proportional].

$\Rightarrow \quad x + iy = 0 \quad \Rightarrow \quad x = 0, y = 0$. Therefore, (D) is the Ans.

31. The event that the fifth toss results in a head is independent of the event that the first four losses result in tails.

$\therefore$ Probability of the required event = 1/2

Therefore, (A) is the Ans.

32. Distinct n digit numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n so that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow \quad n - 2 \geq 5 \qquad \Rightarrow \quad n \geq 7$ so the least value of n is 7.

Therefore, (B) is the Ans.

33. The no. of ways of placing 3 black balls without any restriction is $^{10}C_3$. Since we have total 10 places of putting 10 balls in a row and firstly we will put 3 black balls. Now the no. of ways in which no two black balls put together is equal to the no. of ways of choosing 3 palaces marked—out of eight places.

— W — W — W — W — W — W — W —

This can be done in $^8c_3$ ways. Thus, probability of the required event

$= \dfrac{^8c_3}{^{10}c_3} = \dfrac{8 \times 7 \times 6}{10 \times 9 \times 8} = \dfrac{7}{15}$. Therefore, (B) is the Ans.

34. $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ (given)

Now put $\theta = 0$, we get $\quad 0 = b_0$

$\therefore \quad \sin n\theta = \sum_{r=1}^{n} b_r \sin^r \theta$ is true

$\Rightarrow \quad \dfrac{\sin n\theta}{\sin \theta} = \sum_{r=1}^{n} b_r (\sin \theta)^{r-1}$

taking limit as $\theta \to 0$

$\Rightarrow \quad \lim_{\theta \to 0} \dfrac{\sin n\theta}{\sin \theta} = \lim_{\theta \to 0} \sum_{r=1}^{n} b_r (\sin \theta)^{r-1}$

$\Rightarrow \quad \lim_{\theta \to 0} \dfrac{n\theta \cdot \dfrac{\sin n\theta}{n\theta}}{\theta \cdot \dfrac{\sin\theta}{\theta}} = b_1 + 0 + 0 + 0 \ldots\ldots$

Other values becomes zero for higher powers of $\sin \theta$.

$\Rightarrow \quad \dfrac{n \cdot 1}{1} = b_1$

$\Rightarrow \quad b_1 = n$ Therefore, (B) is the Ans.

35. $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$ (formula)

and $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ (formula)

and $\sin 15° \cos 75° = \sin 15° . \sin 15° = \frac{1}{4}(2-\sqrt{3})$. Therefore, all these values are irrational and

$\sin 15° \cos 15° = \frac{1}{2} \cdot 2 \sin 15° \cos 15° = \frac{1}{2} \cdot \sin 30° = \frac{1}{4}$ which is rational.

therefore, (C) is the Ans.

36. It is given that $x^2 + y^2 = a^2$ ...(1)

and $xy = c^2$ ...(2)

We obtain $x^2 + c^4/x^2 = a^2$

$\Rightarrow \quad x^4 - a^2x^2 + c^4 = 0$ ...(3)

Now x_1, x_2, x_3, x_4 will be roots of (3).

Therefore $\Sigma x_1 = x_1 + x_2 + x_3 + x_4 = 0$

and product of the roots $x_1 x_2 x_3 x_4 = c^4$

Similarly, $y_1 + y_2 + y_3 + y_4 = 0$ and $y_1 y_2 y_3 y_4 = c^4$

Therefore, (a), (b), (c) and (d) are the answers.

37. It is given that $P(E) \le P(F) \Rightarrow E \subseteq F$ ...(1)

and $P(E \cap F) > 0 \Rightarrow E \ne F$ ...(2)

(A) occurrence of $E \Rightarrow$ occurrence of F from (2)

(B) occurrence of $F \Rightarrow$ occurrence of E from (2)

(C) non-occurrence of $E \Rightarrow$ non-occurrence of F from (1)

Therefore, (D) is the Ans.

38. **A.** $\vec{u} \cdot (\vec{v} \times \vec{w})$ is a meaningful operation, therefore, (A) is the Ans.

B. $\vec{u} \cdot (\vec{v} \cdot \vec{w})$ is not meaningful since $\vec{v} . \vec{w}$ is a scalar quantity and for dot product both quantities should be vector. Therefore, (B) is not the Ans.

C. $(\vec{u} \cdot \vec{v})\, \vec{w}$ is meaningful since it is a simple multiplication of vector and scalar quantity. Therefore, (C) is not the Ans.

D. $\vec{u} \times (\vec{v} \cdot \vec{w})$ is not meaningful since $\vec{v} . \vec{w}$ is a scalar quantity and for cross product, both quantity should be vector. Therefore, (D) is the Ans Hence (A) and (C) are the Ans.

39. $\int_0^x f(t)\,dt = x + \int_x^1 t\, f(t)\,dt,$ (given)

Differentiating both sides w.r.t. x, we get

$$f(x) \cdot 1 = 1 - x\, f(x) \cdot 1$$

$$\Rightarrow \quad (1 + x)\, f(x) = 1$$

$$\Rightarrow \quad f(x) = 1/(1 + x)$$

$$\Rightarrow \quad f(1) = \frac{1}{1+1} = \frac{1}{2}$$

Therefore, (A) is the Ans

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$

Differentiate the whole equation w.r.t. x

$$h'(x) = f'(x) - 2f(x) \cdot f'(x) + 3\, f^2(x) \cdot f'(x)$$

$$= f'(x)\,[1 - 2\, f(x) + 3f^2(x)]$$

$$= 3f'(x)\left[(f(x))^2 - \frac{2}{3}\, f(x) + \frac{1}{3}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{1}{3} - \frac{1}{9}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{3-1}{9}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{2}{9}\right]$$

Note that $h'(x) < 0$ if $f'(x) < 0$ and $h'(x) > 0$ if $f'(x) > 0$

Therefore, $h(x)$ is increasing function if $f(x)$ is increasing function, and $h(x)$ is decreasing function if $f(x)$ is decreasing function.

Therefore, (A) and (C) are the Ans.